Math 304 (Spring 2015) - Homework 3

Problem 1.

Compute the following determinants:

$$\begin{vmatrix} 6 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 2 & 1 \end{vmatrix}, \quad \begin{vmatrix} 2 & 3 & 0 & 2 \\ 4 & 3 & 2 & 1 \\ 6 & 0 & 0 & 3 \\ 7 & 0 & 0 & 4 \end{vmatrix}, \quad \begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix}$$

Solution:

$$\begin{vmatrix} 6 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 2 & 1 \end{vmatrix} = 24, \quad \begin{vmatrix} 2 & 3 & 0 & 2 \\ 4 & 3 & 2 & 1 \\ 6 & 0 & 0 & 3 \\ 7 & 0 & 0 & 4 \end{vmatrix} = 18, \quad \begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} = 17$$

Problem 2.

Compute the determinant

$$\begin{vmatrix} 0 & 7 & 5 & 3 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & -1 \\ 1 & 1 & 1 & 2 \end{vmatrix}$$

Solution:

$$\begin{vmatrix} 0 & 7 & 5 & 3 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & -1 \\ 1 & 1 & 1 & 2 \end{vmatrix} = 14$$

Problem 3.

Use the adjoint to find the inverse of the following matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

Solution: Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$.

$$A^{-1} = \frac{1}{\det(A)} \text{adj}(A) = \begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1/3 & 0 \\ -1/9 & -5/18 & 1/6 \end{bmatrix}$$

Problem 4.

Let A be an $n \times n$ matrix and α a scalar. Show that

$$\det(\alpha A) = \alpha^n \det(A).$$

(Hint: can we obtain the matrix (αA) from the matrix A by a sequence of elementary row operations?)

Solution: αA can be obtained from A by elementary row operations of type II. Indeed,

$$\alpha A = E_1 \cdots E_n A$$

where E_i is the corresponding elementary matrix that multiplies the i-th row by the constant α. It follows that

$$\det(\alpha A) = \det(E_1) \cdots \det(E_n) \det(A) = \alpha^n \det(A)$$

Problem 5.

If A and B are 4×4 matrices with $\det(A) = 4$ and $\det(B) = 3$. Find the value

(a) $\det(AB)$

(b) $\det(2A)$

(c) $\det(A^{-1}B)$

Solution:

(a) $\det(AB) = 12$.

(b) $\det(2A) = 2^4 \det(A) = 64$.

(c) $\det(A^{-1}B) = \frac{3}{4}$.